5. The obtained results permit the following recommendations to be made.

If the outer radius of the ring b is not very close to R, then the annular shape of the electrodes ensures a more uniform temperature field than the disk shape. This result corresponds to the qualitative evaluations obtained in the study of the current density distribution for operation with dc. The "anomalous" effect when $b \sim R$ is due to the effect of heat transfer from the surface, which in our case is considerable.

Out of the examined annular electrodes, the greatest "uniformity" with optimal current is ensured by variant 2 which corresponds to the "mean position" of the ring: $a = R/2$, $b = R/\sqrt{2}$.

Table 1 also presents the current densities on the electrodes. It can be seen that the current density competes with the level of uniformity of the field, and in the variant that is optimal from the point of view of uniformity of the field the current density is greatest although it does not exceed the permissible values.

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REGULAR REGIME IN TRANSLUCENT MATERIALS

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The article analyzes radiative—conductive heat transfer in a translucent plate. It is established that a regime exists which is analogous to the regular regime in conductive heat transfer.

The question of the existence of regular regimes in translucent materials was dealt with by several authors [1-4] but the results of [1, 2] differed substantially from those of [3, 4]. The authors of the last two articles explained this difference by stating that in [1, 2] the conductive component of heat transfer was predominant whereas in [3, 4] it was the radiative component. However, the authors of [1, 2] investigated the regular regime with radiative and convective heat transfer acting in the same direction (radiative and convective heating or radiative and convective cooling) whereas in [3, 4] these components were opposed to each other (convective heating and radiative cooling). This required additional theoretical investigation of the regular regime in a translucent plate where the radiative component of heat transfer is equal to or larger than the conductive component, and both act in the same direction.

The difference algorithm for investigating nonsteady radiative—conductive heat exchange was constructed in the following manner: the energy equation for an infinitely thin layer of a translucent plate with optically smooth surfaces, separated from the opaque surfaces by a medium conducting thermal radiation, can be expressed in the form [5]

$$C_\gamma \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\Lambda \frac{\partial T}{\partial x}\right] - \frac{\partial q^r}{\partial x},\ 0 \leqslant x \leqslant b, \tag{1}$$

where $\partial q^r/\partial x$ is determined by the expression

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$$\frac{\partial q^r}{\partial x} = 2\int\limits_{\lambda=0}^{\infty} d\lambda \int\limits_0^1 \varkappa_\lambda \{2n_\lambda^2 I_\lambda(T) - \pi [I_\lambda^+(\tau_\lambda, \beta) + I_\lambda^-(\tau_\lambda, \beta)]\} \mu d\mu$$

with the initial and boundary conditions

$$T(x, t=0) = T^0(x), \tag{1a}$$

$$\Lambda \frac{\partial T}{\partial x}\Big|_{x=0(b)} = \alpha_{1(2)}(T_{\mathrm{md},1(2)} - T_{g,1(2)}) + \pi \int\limits_{\lambda_{\mathrm{op}}=0}^{\infty} [I_\lambda(T_{g,1(2)}) - I_\lambda(T_{g,1(2)})]\, d\lambda \Big/ \left[\frac{1}{\varepsilon_{1(2)}(\lambda)} + \frac{1}{\varepsilon(\lambda)} - 1\right]. \tag{1b}$$

The magnitude ∂q^r in expression (1) is the differential of the flux density of the resulting radiation. This can be represented as the increase of density of the resulting flux in case of emission of radiation by the layer dx and absorption by it of radiation from the entire bulk of the translucent plate and the opaque surfaces with a view to the multiple reflections from the surfaces of the plate and from the opaque surfaces. In this case the entire path of the radiation from emission to complete absorption with intermediate reflections is traced; therefore there is no need to use the equations of transfer of the radiation and the boundary conditions for these equations.

The multiple reflections of radiation between the translucent and the opaque surfaces were taken into account with the aid of the effective reflection factor [2]

$$\overset{e}{\rho}_{1(2)}(\lambda, \beta) = \rho(\lambda, \beta) + \rho_{1(2)}(\lambda, \beta)\ \frac{[1-\rho(\lambda, \beta)]^2}{1-\rho_{1(2)}(\lambda, \beta)\rho(\lambda, \beta)}, \tag{2}$$

where the dependence of the reflection factors of the translucent surface $\rho(\lambda, \beta)$ on the angle β is taken into account by Fresnel's formulas [5].

Using the Bouguer—Lambert law of absorption, we can write the expression determining the fraction of radiation absorbed by the layer dx, which propagates with intensity $I_{\lambda,0}(\beta)$ from the surface of the plate to the inside at the angle β, in the form

$$dI_\lambda(\beta) = I_{\lambda,0}(\beta)\{\exp(-\tau_\lambda/\mu) - \exp[-(\tau_\lambda + d\tau_\lambda)/\mu]\}. \tag{3}$$

To establish the dependence of the intensity of the radiation propagating in the plate within the limits of the angle $0 \leqq \beta \leqq \pi/2$ on the direction, we divide this angle into K parts, and in each of them the reflection factor of the surface of the translucent material is averaged:

$$\Delta\beta_1 + \Delta\beta_2 + \ldots + \Delta\beta_K = \pi/2,$$

where $\Delta\beta_1 = \beta_1$, $\Delta\beta_2 = \beta_2 - \beta_1$; ...; $\Delta\beta_K = \pi/2 - \arcsin(1/n_\lambda)$. Here, $\Delta\beta_K$ is the angle within which the reflection factor of the internal radiation from the surface is equal to unity.

Expression (3) for the k-th solid angle $2\pi \sin\beta_k \Delta\beta_k$ is written in the form

$$\Delta q_{\lambda,k} = 2\pi I_{\lambda,0,k} \int\limits_{\mu_k}^{\mu_{k-1}} \mu\left[\exp\left(-\frac{\tau}{\mu}\right) - \exp\left(-\frac{\tau+\Delta\tau}{\mu}\right)\right] d\mu.$$

Using the substitution $\mu' = \mu/\mu_{k(k-1)}$, we obtain

$$\Delta q_{\lambda,k} = 2\pi I_{\lambda,0,k}\left\{\mu_{k-1}^2 \int\limits_0^1 \mu'\left[\exp\left(-\frac{\tau_\lambda}{\mu'\mu_{k-1}}\right) - \exp\left(-\frac{\tau_\lambda+\Delta\tau_\lambda}{\mu'\mu_{k-1}}\right)\right] d\mu' - \right.$$
$$\left. - \mu_k^2 \int\limits_0^1 \mu'\left[\exp\left(-\frac{\tau_\lambda}{\mu'\mu_k}\right) - \exp\left(-\frac{\tau_\lambda+\Delta\tau_\lambda}{\mu'\mu_k}\right)\right] d\mu'\right\}$$

and, finally,

$$\Delta q_{\lambda,k} = \pi I_{\lambda,0,k}[E_k(\tau_\lambda) - E_k(\tau_\lambda+\Delta\tau_\lambda)](\mu_{k-1}^2 - \mu_k^2), \tag{4}$$

where $I_{\lambda,0,k}$ is the intensity of the monochromatic radiation propagating from the surface of the plate into the bulk and averaged within the limits $\Delta\beta k$:

$$E_k[\tau_\lambda(\tau_\lambda+\Delta\tau_\lambda)] = 2\{\mu_{k-1}^2 E_3[\tau_\lambda(\tau_\lambda+\Delta\tau_\lambda)/\mu_{k-1}] - \mu_k^2 E_3[\tau_\lambda(\tau_\lambda+\Delta\tau_\lambda)/\mu_k]\}/(\mu_{k-1}^2-\mu_k^2). \quad (5)$$

Here $E_3(\tau_\lambda/\mu) = \int_0^1 \mu' \exp(-\tau_\lambda/\mu'\mu)\,d\mu$ is an integroexponential function of third order, determined, e.g., after [5].

The dependence of the absorption coefficient and of the refractive index on the wavelength is taken into account by an exponential approximation of the absorption spectrum of the material with averaging of the mentioned values at each j-th step ($j = 1, 2, \ldots, m$). Thus the expression for the density of the radiation flux emitted by an isothermal layer of translucent material with thickness Δx in the k-th solid angle and the j-th interval of wavelengths has the form

$$\Delta q_j = 2\sigma n_j^2 T^4[1-E_k(\Delta\tau_j)](\mu_{k-1}^2-\mu_k^2)F_\Delta^j. \quad (6)$$

For the sake of lucidity we will consider symmetrical heat transfer with absorption factors and conductive thermal conductivity independent of the temperature. We divide the thickness of the translucent plate into N theoretical layers ($i = 1, 2, 3, \ldots, N$), but it is advisable to take an even number of layers. Then for the i-th theoretical layer we can write the explicit two-layer difference analog of Eq. (1):

$$C\gamma\frac{\Delta x}{\Delta t}(T_i^{l+1}-T_i) = q_1+q_2+q_3-q_4, \quad (7)$$

where

$$q_1 = \frac{\Lambda}{\Delta x}(T_{i+1}-2T_i+T_{i-1});$$

$$q_2 = \sigma T_{\mathrm{op}}^4\sum_{j=1}^{m}F_\Delta^j\sum_{k=1}^{K-1}(1-\rho_{k,j}^{\mathrm{e}})(\mu_{k-1}^2-\mu_k^2)[E_k(\tau_{j,i-0,5}) -$$

$$- E_k(\tau_{j,i+0,5}) + E_k(\tau_{j,N}-\tau_{j,i+0,5}) - E_k(\tau_{j,N}-\tau_{j,i-0,5})]/[1-\rho_{k,j}^{\mathrm{e}}E_k(\tau_{j,N})];$$

$$q_3 = \sigma\sum_{j=1}^{m}n_j^2F_\Delta^j\sum_{f=1}^{N/2}T_f^4\{[1-E(\Delta\tau_j)]\Big|E|\tau_{j,i-0,5}-$$

$$-\tau_{j,f}|-E|\tau_{j,i+0,5}-\tau_{j,f}|\Big|+\Big|E|\tau_{j,i-0,5}-\tau_{j,N+1-f}|-$$

$$-E|\tau_{j,i+0,5}-\tau_{j,N+1-f}|\Big|+\sum_{k=1}^{K}\frac{[1-E_k(\Delta\tau_j)](\mu_{k-1}^2-\mu_k^2)\rho_{k,j}^{\mathrm{e}}}{1-\rho_{k,j}^{\mathrm{e}}E_k(\tau_{j,N})}\times$$

$$\times[E_k(\tau_{j,i-0,5})-E_k(\tau_{j,i+0,5})+E_k(\tau_{j,N}-\tau_{j,i+0,5})-$$

$$-E_k(\tau_{j,N}-\tau_{j,i-0,5})][E_k(\tau_{j,N}-\tau_{j,f})+E_k(\tau_{j,f})]\},$$

$$q_4 = 2\sigma T_i^4\sum_{j=1}^{m}n_j^2F_\Delta^j[1-E(\Delta\tau_j)]F_\Delta^j.$$

In expression (7), q_1 is the density of the conductive thermal flow from the layers adjacent to the theoretical layer; q_2 is the density of the radiative thermal flow absorbed by the i-th layer with a view to the multiple reflections from the surfaces of the plate and from the opaque surfaces. The numerator in the expression q_2 determines the absorption of radiation in its single passage through the plate, where the expression $E_k(\tau_{j,i-0.5}) - E_k(\tau_{j,i+0.5})$ in accordance with formula (4) takes into account the absorption of radiation from the first opaque surface by the i-th layer, expression $E_k(\tau_{j,N}-\tau_{j,i+0.5}) - E_k(\tau_{j,N}-\tau_{j,i-0.5})$ from the second opaque surface, and the denominator takes into account absorption in multiple passages through the plate after reflections from the interfaces. The first part of q_3 determines the radiation from other theoretical layers (denoted by the subscript f) absorbed by the i-th layer before being reflected from the boundaries of the plate, and since

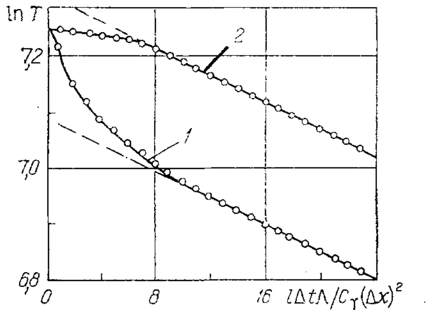

Fig. 1. Change of the temperature of the surface (1) and of the center (2) of a glass plate as a function of time.

the radiation is isotropic, the distribution of the radiation density according to direction is not taken into account. Here, expression $T_f^4\,[1 - E(\Delta\tau_j)]$ in accordance with formula (6) determines the amount of energy radiated by the f-th layer and the (N + 1 − f) layer symmetrical to it, expression $|E|\tau_{j,i-0.5} - \tau_{j,f}| - E|\tau_{j,i+0.5} - \tau_{j,f}||$ determines the amount of energy from the f-th layer absorbed by the i-th layer, and expression $|E|\tau_{j,i-0.5} - \tau_{j,N+1-f}|\ E|\tau_{j,i+0.5} - \tau_{j,N+1-f}||$ determines the energy from the (N + 1 − f) layer absorbed by the i-th layer. The second part of q_3 is the radiation absorbed by the i-th layer from all the theoretical layers including the i-th layer, after multiple reflections from the boundaries (multiple absorption is taken into account by the denominator as in the expression for q_2). The magnitude of q_4 determines the energy radiated by the i-th layer (part of this energy, absorbed after multiple reflections by the layer itself, is taken into account in the second part of q_3).

The effect of convective and radiative (in the range of opaqueness of the material of the plate to thermal radiation) heat transfer on the temperature of the calculated boundary layers was determined by the following difference analog of the boundary conditions (1b):

$$T_0 = T_1 + q_5\Delta x/\Lambda, \tag{8}$$

$$T_g = T_1 + 0.25\,(T_0^{l-1} - T_2^{l-1}), \tag{8a}$$

$$q_5 = \alpha\,(T_{md} - T_g) + \sum_{i_{op}=1}^{m_{op}} \frac{\sigma\,(T_g^4 - T_{op}^4)\,F_\Delta^{i_{op}}}{1/\varepsilon_{op,i_{op}} + 1/\varepsilon_{i_{op}} - 1}. \tag{8b}$$

This schema, compared with Schmidt's schema [6], which is used as a rule, is distinguished by the more accurate determination of the surface temperature which also corresponds to the physical essence of the process of nonsteady heat transfer because in the determination of the surface temperature not only the mean temperature of the calculated boundary layer is taken into account, but also the temperature gradient in this layer.

Let us consider symmetrical cooling of a plate of glass with optically smooth surfaces, with constant temperature of the air stream flowing around the plate, and opaque interfaces. The initial data for calculating according to the algorithm (7)-(8b) with consideration of [3, 4] were taken as follows: $b = 2\cdot10^{-2}$ m, $\Lambda = 1.65$ W/m·deg, $n = 1.5$, $C\gamma = 2.5$ J/m³·deg, $\alpha = 125$ W/m²·deg, $\varepsilon_{i,j_{op}} = 0$, $\varepsilon_{op,j,k} = 1$, $\varkappa(0.25\text{-}4.8\ \mu m) = 50\ m^{-1}$; $T(x, t = 0) = 1400$°K; $T_{op} = T_{md} = 293$°K.

Figure 1 presents the results of the calculation of radiative—conductive heat transfer with these data. Since heat transfer was calculated for constant temperatures of the opaque surfaces and of the medium, the dependence of the logarithms of the temperatures (dots: calculated values) on this time are presented in accordance with the theory of regular regime [7] instead of the dependence of the temperatures of the surface and of the center of the plate on the dimensionless time $\ell\Delta t\Lambda/C_\gamma(\Delta x)^2$. The temperature curves in Fig. 1 are bounded by the instant corresponding to a surface temperature of 900°K because when the plate is further cooled, its temperature field remains self-similar in time.

It can be seen from Fig. 1 that the period of a nonordered nonsteady process is followed by the regular regime of first kind (the dashed lines through the calculated values of the logarithms of the temperatures of the surface and of the center of the plate are parallel to each other).

The ratio between the conductive and the radiative components of heat transfer was evaluated by the ratio of the density of the conductive thermal flow to the density of the radiative thermal flow in the approximation of the diffusion of the radiation with a temperature jump as boundary condition (the results of the calculations by this approximation are in good agreement with the exact solution in the entire range of optical densities [5]):

$$\frac{q_{\mathrm{cond}}}{q_{\mathrm{rad}}} = \frac{\Lambda \Delta T / b}{\sigma n^2 \Delta (T^4) F_{\Delta} \Big/ \left(\frac{3}{4} \varkappa b + \frac{1}{\varepsilon_{1,\mathrm{e}}} + \frac{1}{\varepsilon_{2,\mathrm{e}}} - 1 \right)}, \tag{9}$$

where F_{Δ} and $\varkappa$ relate to the range of wavelengths from 0.25 to 4.8 μm. We linearize the function T^4 by expansion into a Taylor series, discard the terms of the expansion of higher orders, and change expression (9) into the form

$$\frac{q_{\mathrm{cond}}}{q_{\mathrm{rad}}} = \frac{\Lambda \left(0.75 \varkappa b + \frac{1}{\varepsilon_{1,\mathrm{e}}} + \frac{1}{\varepsilon_{2,\mathrm{e}}} - 1\right)}{4 \sigma n^2 \bar{T}^3 F_{\Delta} b}, \tag{10}$$

where $\bar{T}$ is the mean temperature over the cross section of the plate.

The results presented in Fig. 1 express the state of the plate on the section of cooling on which the radio $q_{\mathrm{cond}}/q_{\mathrm{rad}}$ is smaller than 0.25, i.e., the radiative component of heat transfer is more than four times larger than the conductive component. These results differ substantially from the results of [3, 4] where the radiative component of heat transfer was also predominant. It seems that the differently directed action of the convective and the radiative components in the investigation of radiative–conductive heat exchange under conditions of regular regime, which occurred in this work, makes the onset of the regular regime difficult.

The results obtained above show that in the investigation of regular regimes in translucent materials it is indispensable to take into account the relative mutual direction of the convective and the radiative components of heat transfer.

NOTATION

q^r, flux density of the resulting radiation; λ, wavelength at which the material of the plate is translucent to radiation; λ_{op}, wavelength at which the material of the plate is opaque to radiation; $I_{\lambda}^{\pm}(\tau_{\lambda}, \beta)$, intensity of monochromatic radiation in the direction of the angle β at the point with the optical coordinate $\tau_{\lambda} = \varkappa_{\lambda} x$ propagating in the positive or negative direction of the x axis; $I_{\lambda}(T_{g1(2)})$ and $I_{\lambda}(T_{\mathrm{op}1(2)})$, Planck's function for the temperature of the surfaces of glass and for opaque surfaces; T_{md}, temperature of the gaseous medium; $\varkappa_{\lambda}$, Λ, C_{γ}, and n_{λ}, absorption coefficient, thermal conductivity, volume heat capacity, and refractive index, respectively, of the translucent material; $\rho_{1(2)}(\lambda, \beta)$ and $\rho(\lambda, \beta)$, reflection factors of opaque surfaces and of the surface of the plate, respectively; $\varepsilon_{1(2)}(\lambda)$ and $\varepsilon(\lambda)$, hemispherical emissivities of the opaque surfaces and of the surface of the plate, respectively; Δx and Δt, transverse and time steps, respectively, of the calculation network; α, coefficient of convective heat transfer between the plate and the gaseous medium washing it; F_{Δ}^{j}, fraction of the intensity of absolutely black radiation per interval of wavelengths with mean absorption coefficient $\varkappa_j$ and refractive index n_j in stepped approximation of the transparency range of the absorption spectrum of the material of the plate (m is the number of steps); $F_{\Delta\mathrm{op}}^{j}$, fraction of the intensity of absolutely black radiation in the opacity range of the material of the plate per interval of wavelengths with mean refractive index $n_{j\mathrm{op}}$ (m is the number of steps); σ, Stefan–Boltzmann constant; $E(\tau) = 2E_3(\tau)$ and $E_k(\tau)$, functions determining the fraction of radiation transmitted by a layer of translucent material with optical thickness τ within the limits of the angles $0-\pi/2$ and $\Delta\beta_k$, respectively; T_i, $T_i^{\ell-1}$ and $T_i^{\ell+1}$, temperature of the i-th layer at the ℓ-th and $(\ell \mp 1)$-th instant; T_0, temperature of a fictitious point situated at the ℓ-th instant at a distance of $0.5\Delta x$ from the surface of the plate; $\tau_{j,N} = \varkappa_j b$, optical thickness of the plate; b, thickness of the plate; $\rho_{k,j}^{e}$, effective reflection factor averaged within the limits of $\Delta\beta_k$, taking into account multiple reflections of the radiation between the translucent surface and the opaque surface; $\varepsilon_{\mathrm{op},j\mathrm{op}}$ and $\varepsilon_{j\mathrm{op}}$, hemispherical emissivities of the opaque surface and of the translucent surface in the range of opaqueness of the material of the plate, averaged over $\Delta\lambda$.

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NUMERICAL ANALYSIS OF TRANSPORT PHENOMENA IN SEMICONDUCTOR DEVICES AND STRUCTURES.

3. MODELING OF MIS STRUCTURES

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A universal algorithm for multidimensional numerical analysis of unipolar semiconductor devices is studied.

The theoretical study of unipolar semiconductor devices is at the present time most often carried out employing numerical models based on the solution of the equations of continuity for holes and electrons and Poisson's equation for the electrostatic potential [1].

It is sufficient to cite only some works on the multidimensional analysis of MOS transistors with short channels. Thus in [2, 3] the mechanisms of avalanche breakdown were studied; in [4] the effect of the spread in a number of the electrophysical parameters (channel length, impurity concentration in the substrate, depth of the p—n junction, etc.) on one of the basic parameters — the threshold voltage — was studied; in [5, 6] the effect of adjoining was studied, etc. Naturally, the importance of such studies increases with the transition to the submicron technology for fabricating integrated circuits because of the complexity of the experimental development of such circuits.

One of the basic difficulties standing in the way of the assimilation of numerical experiments in practice is the lack of efficient and reliable universal algorithms for the multidimensional numerical analysis of unipolar semiconductor devices. Thus the most efficient algorithms and programs [7, 8], based on Mock's method [9], do not permit carrying out a rigorous calculation of devices in prebreakdown operating states [3] and taking into account the mechanisms of surface recombination [10]. This is linked with the fact that in Mock's method [9] it is assumed that there is no recombination-generation term in the equation of continuity. Algorithms which do not have this drawback either make use of additional physical assumptions [11] or they require supercomputers [12] or they require dense grids in the neighborhood of the insulator—semiconductor interface [13]. The latter circumstance, naturally, places the problem of selecting a grid at the forefront [14].

In this work we propose a universal algorithm for the multidimensional numerical analysis of the static states of unipolar semiconductor devices, based on the method of [1] and not having the above-mentioned drawbacks. The efficiency of the algorithm is illustrated for the

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